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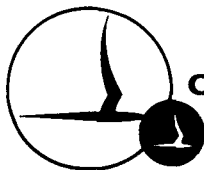
AS AD No.

## UNSTEADY STALL OF AXIAL FLOW COMPRESSORS

Prepared for:  
U.S. ARMY  
HARRY DIAMOND LABORATORIES  
WASHINGTON 25, D.C.

### SECOND QUARTERLY PROGRESS REPORT

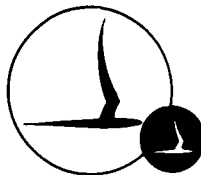
By: W.G. Brady, G.R. Ludwig  
Contract No. DA 49-186-AMC-13(X)  
CAL Report No. AM-1762-S-2  
June 1963



**CORNELL AERONAUTICAL LABORATORY, INC.**

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BUFFALO 21, NEW YORK

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## TABLE OF CONTENTS

	<u>Page</u>
I SUMMARY	1
II INTRODUCTION	2
III WORK ACCOMPLISHED	3
A. Rotating Cylinder Experiments	3
B. Theoretical Results	11
IV WORK FOR THE NEXT QUARTER	15
V REFERENCES	16
DISTRIBUTION	

# LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Hot Wire Probe Calibrations (Two-Wire Mode of Operation)	4
2	Revised Notation for Boundary Layer on Rotating Cylinder (Supercedes that of Figure 1(b) of Reference 1)	5
3	$y$ Vs. $u/U$ ; $\Theta = 100^\circ$ ; $\Omega = -513$ RPM	7
4	Boundary-Layer Velocity Profiles and Thicknesses Vs. $\Theta$ ; $U_\infty \approx 30$ fps, $\Omega = 0$ RPM	8
5	Boundary-Layer Velocity Profiles and Thicknesses Vs. $\Theta$ ; $U_\infty \approx 30$ fps, $\Omega \approx +500$ RPM	9

## I SUMMARY

This report discusses the work accomplished during the second quarter (March - May 1963) on "Research on Unsteady Stall of Axial Flow Compressors", Contract No. DA 49-186-AMC-13(X), for the Harry Diamond Laboratories, Washington, D. C., by Cornell Aeronautical Laboratory, Inc.

Initial effort in this program is concentrated on continuation of previous research at CAL concerning (1) an experimental study of the separation of a laminar boundary layer from walls moving upstream and downstream in a steady flow using a shrouded rotating cylinder in a wind tunnel, and (2) an analytical study of compatibility conditions between circulation disturbances on an actuator disk in a mean flow with swirl and the appearance of vorticity downstream of the disk.

Hot wire anemometer equipment previously on order was received. This equipment was modified to provide proper temperature compensation for use with nonstandard probes. Boundary layer velocity profiles were obtained over approximately half the range of angular positions on the cylinder which appear to be necessary in order to define separation velocity profiles for upstream- and downstream-moving walls.

The analysis for compatibility conditions governing the existence of rotating circulation disturbances on a single-stage axial-flow compressor disk was continued. The very severe restrictions imposed on the results previously reported (radial perturbation velocity component assumed equal to zero, no radial variation in flow variables) were relaxed to the extent that only the assumption that the radial perturbation velocity component was assumed zero. Under this assumption and imposing the conditions that the disturbance flow be bounded with time and axial distance from the compressor disk, it was found that, as before, the only possible rotating circulation disturbance on the compressor disk is one rotating with the blades.

## II INTRODUCTION

Rotating stall has been a continuing problem for the designers of axial-flow turbojet engines since the inception of their use in aircraft. For the compressor designer, the major problem associated with rotating stall has been one of blade fatigue failure resulting from blade resonance with the passage of stall zones. From a performance point of view, unsteady stall phenomena may place severe limitations on the acceleration or deceleration of a jet engine.

Extensive research effort has been devoted to this problem during the past two decades. Progress was made in that a degree of understanding of the phenomenon was attained; however, the basic problem remained unsolved when lack of funds reflecting a declining interest in jet engine development brought this research effort nearly to a halt in the late 1950's. In particular, the unsteady boundary-layer separation phenomena which, presumably, underlie rotating stall of compressor elements continue to pose problems of fundamental interest.

Due to a revival of interest in axial-flow jet engine development, this program was initiated to continue a methodical approach to solutions for the problems posed by unsteady compressor stall. The work reported here is a continuation of a research program concerning rotating stall in axial-flow compressors, carried out by the Cornell Aeronautical Laboratory over a three-year period ending in January 1959. The ultimate objective of this research effort is an increased understanding of rotating stall and other unsteady surge characteristics of axial compressors. Included will be studies of the aerodynamics of compressor stages, studies of unsteady sectional-lift characteristics of single-blade elements near stall, and studies of possible methods of alleviating unsteady surge characteristics. In the current year's effort, the emphasis is on the continuation of two studies started under the original program: (1) a theoretical study of the role of entering swirl pattern in causing rotating stall of a single stage, and (2) an experimental study of the separation criteria for a moving surface (specifically a shrouded rotating cylinder).



### III WORK ACCOMPLISHED

#### A. Rotating Cylinder Experiments

Details of the motivation of this work and of the basic experimental apparatus being employed were reported in the First Quarterly Progress Report (Reference 1) and will not be repeated here.

#### Hot Wire Anemometer Calibration

The Flow Corporation hot wire air velocity meter, Model 55B1, ordered during the first quarter (purchased with CAL funds) was delivered early in the period. This unit was purchased with an adapter (Model A-3) which permitted the use of a standard Flow Corporation HWP-A probe in single-wire operation with the meter in place of the special two-wire probe supplied with the Model 55B1 unit (HWP-A probes were available at CAL from the previous rotating stall research program). This adapter was modified to permit use of two HWP-A probes in two-wire operation (for temperature compensation) after preliminary calibration of the single-wire unit showed excessive scatter. After some experimentation with methods for mounting wires on the probes to obtain good solder joints, consistently good calibration data were obtained. Figure 1 shows calibration curves for the first three hot-wires used during the boundary-layer velocity surveys.

Preliminary boundary-layer velocity profile data obtained with an HWP-A probe indicated considerably thinner boundary-layer on the rotating cylinder than the data previously obtained with a total-head probe. The total-head probe data is probably not reliable for the inner portions of the boundary layer near the cylinder surface where displacement effects and large shear gradients would tend to distort the velocity profile. Such data should be reliable, however, near the outer edge of the boundary layer. It was concluded that the HWP-A probes were causing flow interference in the boundary layer. Therefore, a special boundary-layer probe was purchased (Flow Corporation Model HWP-D). Velocity profiles obtained with this probe compared well with total-head probe data for the outer portion of the boundary layer. All boundary-layer data reported here have been obtained with this boundary-layer probe.

#### Boundary-Layer Velocity Profiles

Figure 2 shows the notation appropriate to the boundary-layer velocities (note that this notation is different from the notation shown in Figure 1(b) of Reference 1).

$U_{\infty}$  = the free-stream velocity.

$U$  = the outer-flow velocity on the edge of the boundary layer.

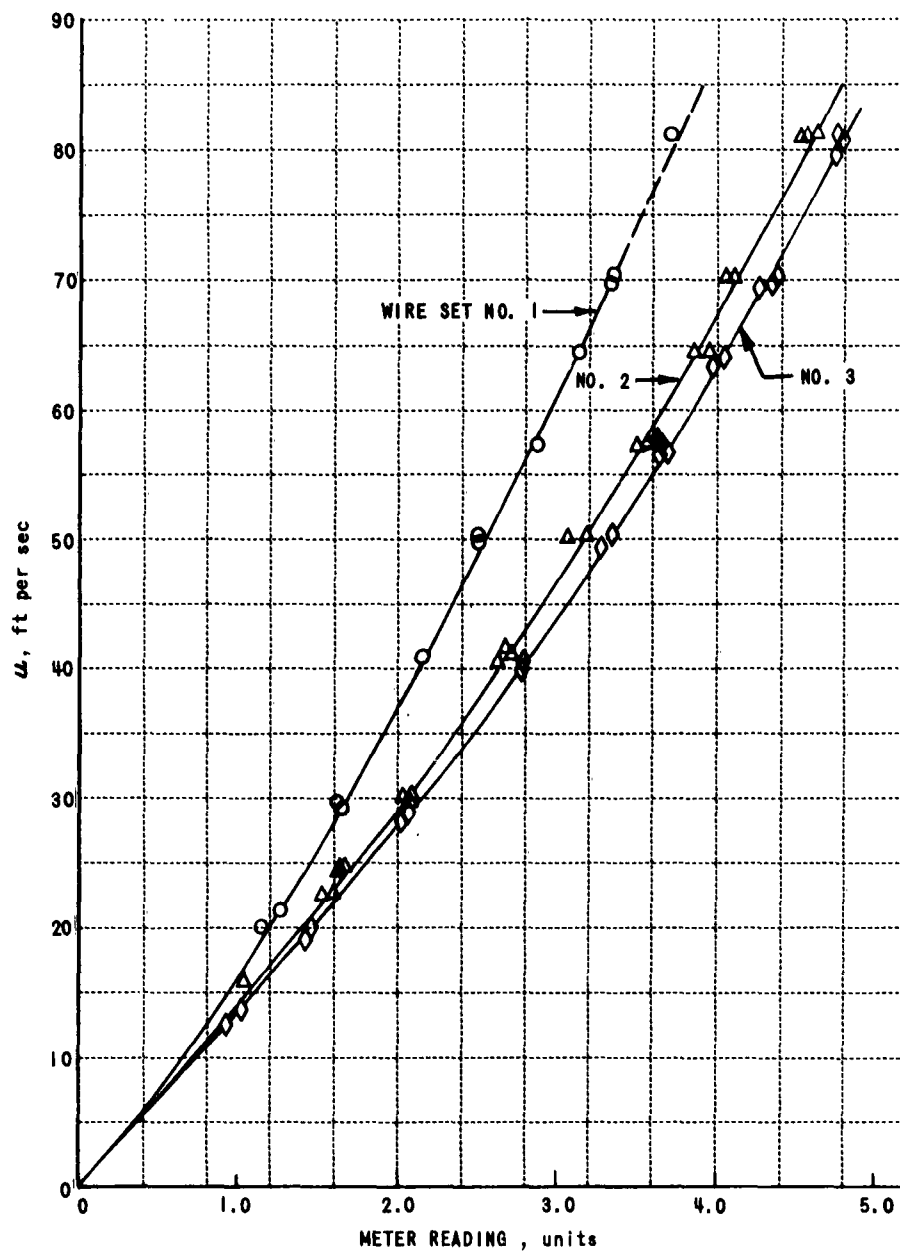


Figure 1 HOT WIRE PROBE CALIBRATIONS  
(TWO-WIRE MODE OF OPERATION)

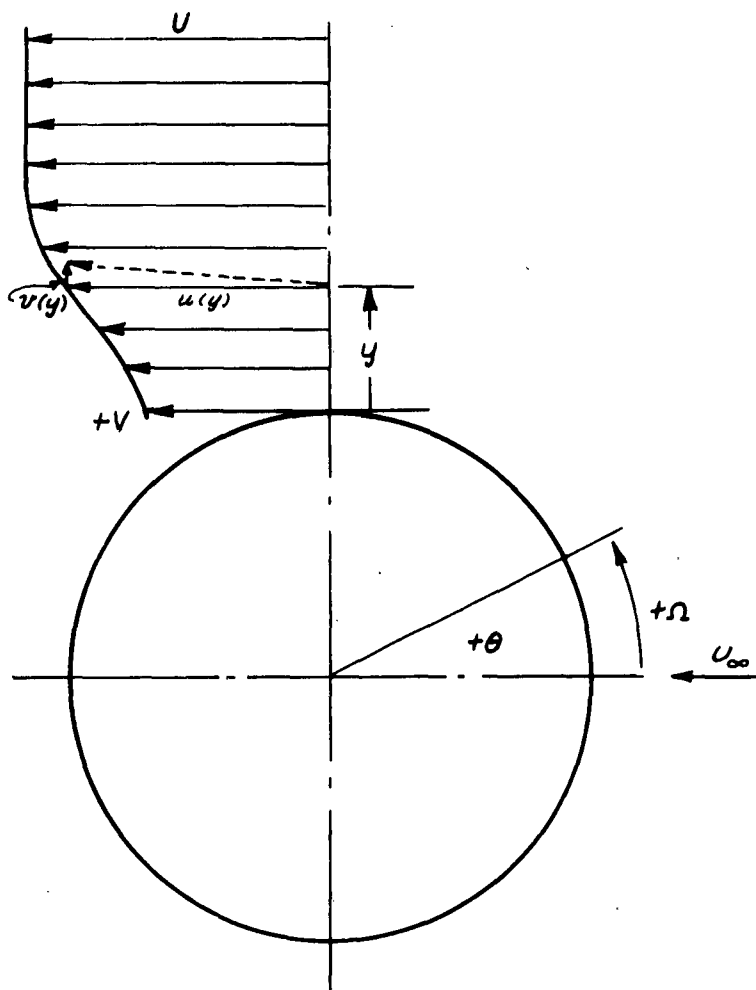


Figure 2 REVISED NOTATION FOR BOUNDARY LAYER ON ROTATING CYLINDER  
(SUPERCEDES THAT OF FIGURE 1(b) OF REFERENCE 1)

$u(y)$  = boundary layer velocity component parallel to the local tangent to the cylinder surface.

$v(y)$  = boundary layer velocity component perpendicular to the local tangent to the cylinder surface.

Velocity profiles in the boundary layer on the rotating cylinder have been obtained in the range of  $\theta$  from  $78^\circ$  to  $120^\circ$  in increments of  $\Delta\theta \approx 10^\circ$  for  $\Omega = \pm 2000, \pm 1500, \pm 1000, \pm 500, \pm 200$  and 0 rpm.

The hot-wire probe is affected by immediate proximity to the surface of the cylinder (within 0.010 inch). This effect is apparently associated with heat transfer between the hot wire and the cylinder surface. It was found possible to correct the experimental data to account for this wall proximity effect, and the correction appears to be negligible for most of the data, becoming of mildly significant magnitude only very close to the wall at  $\theta = 120^\circ$  and  $\Omega = 0$  and -200 rpm.

A further difficulty is posed by the fact that the hot wire measures the magnitude of the velocity vector normal to the wire; hence, in the region of the boundary layer where velocities parallel to the cylinder surface approach zero, the (very small) velocity component perpendicular to the cylinder surface,  $v(y)$ , is recorded by the hot wire. It is in this area that the data are of most interest and importance as regards the velocity profiles at separation. There appears to be no way to experimentally determine the magnitude of this component with the existing equipment, nor is there available theory for moving-wall boundary layers of use in estimating the magnitude of the vertical velocity component near separation. To overcome this difficulty, various assumptions on the shape of the  $v(y)$  profile have been used to correct the experimental data. The results are illustrated in Figure 3 for  $\theta = 100^\circ$  and  $\Omega = -513$  rpm. It is apparent that the corrected profile is relatively insensitive to the shape of the assumed profile for  $v(y)$ . This is generally true of all the boundary-layer data obtained to date. Thus, it can be hoped that the corrected profiles are good approximations to the true profile of the tangential velocity component in the boundary layer.

Velocity profiles for 0, +500 and -500 rpm are shown in Figures 4, 5 and 6. In Figure 4, the 0 rpm velocity profiles behave as expected with an apparent separation point ( $\partial u / \partial y_{y=0} \approx 0$ ) at  $\theta \approx 120^\circ$ . Velocity profiles for  $\theta > 120^\circ$  have not as yet been obtained. The boundary-layer profiles in Figure 5 are very thin (note the expanded scale for  $y$ ), and there is no indication of separation being approached at the maximum value of  $\theta$ . The profiles shown in Figures 4 and 5 contain no correction for  $v(y)$ .

The profiles obtained at -500 rpm (Figure 6) have been corrected for the perpendicular velocity component by assuming that  $v(y) = \text{constant}$  and is equal in magnitude to the minimum velocity recorded by the hot wire. The boundary layer in this case grows much faster than the 0 rpm case, but there is no indication of an abrupt increase in thickness which would indicate

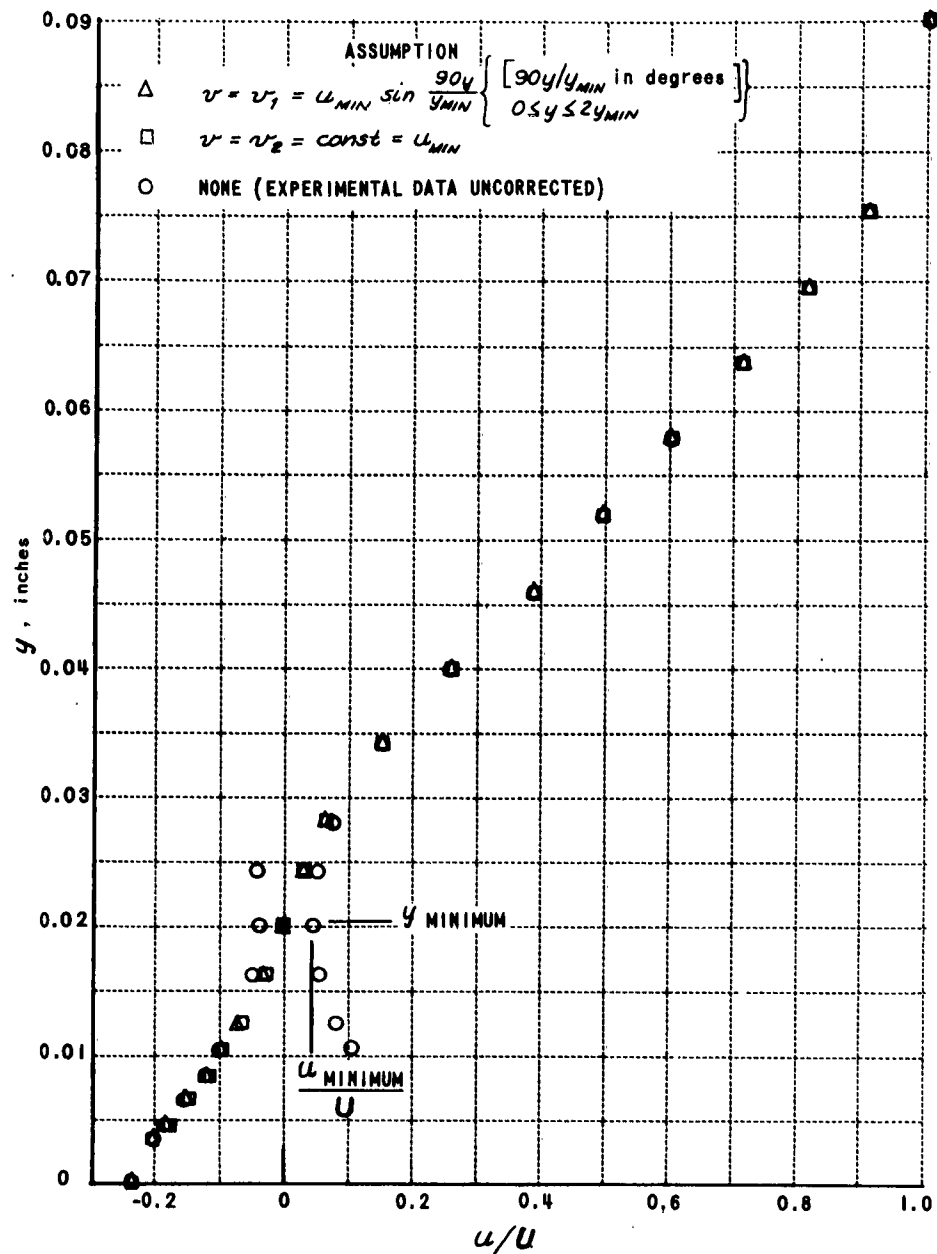


Figure 3  $y$  vs  $u/U$ ;  $\theta = 100^\circ$ ;  $\Omega = -513$  RPM

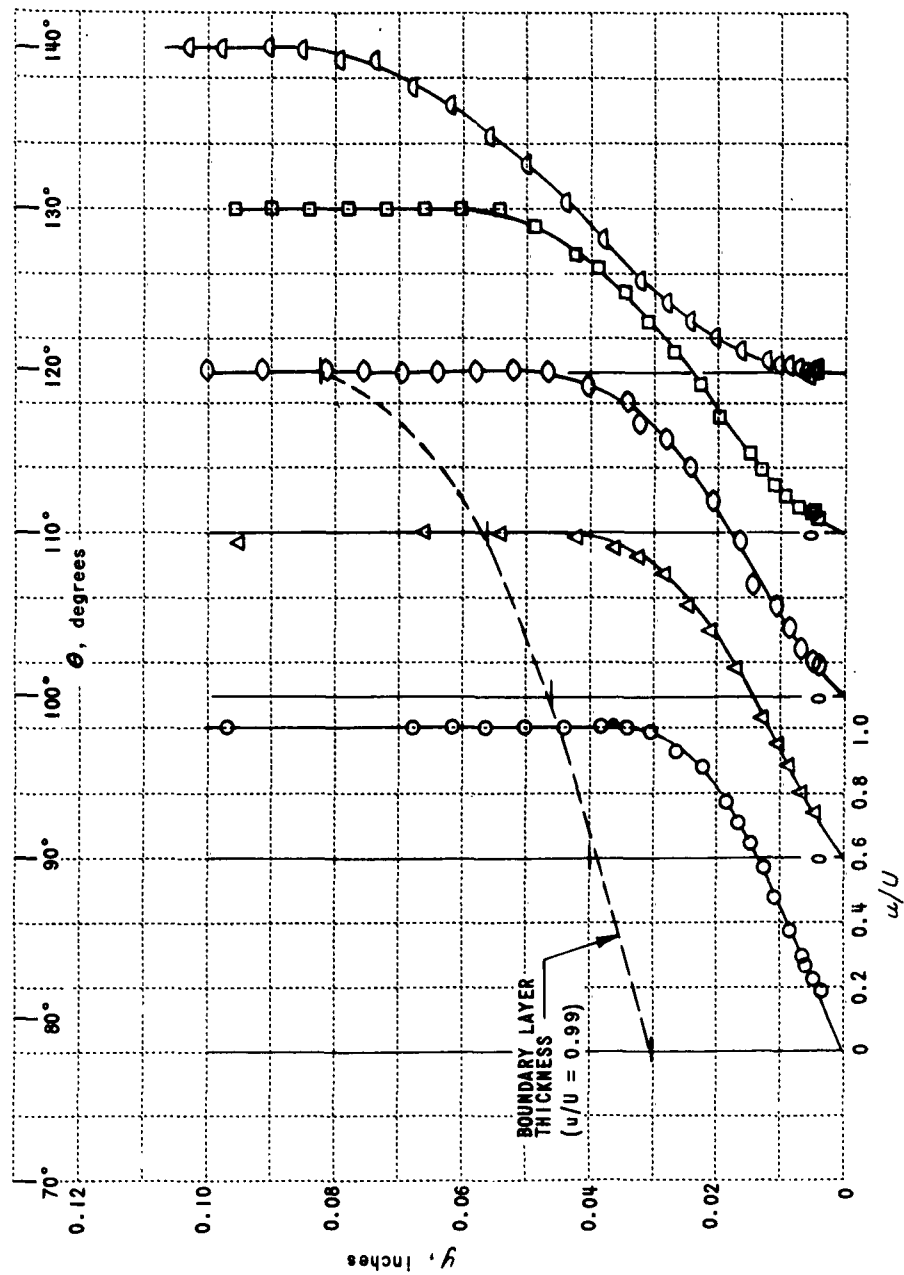


Figure 4 BOUNDARY-LAYER VELOCITY PROFILES AND THICKNESSES vs  $\theta$   
 $U_\infty \approx 30$  fps,  $\Omega = 0$  RPM

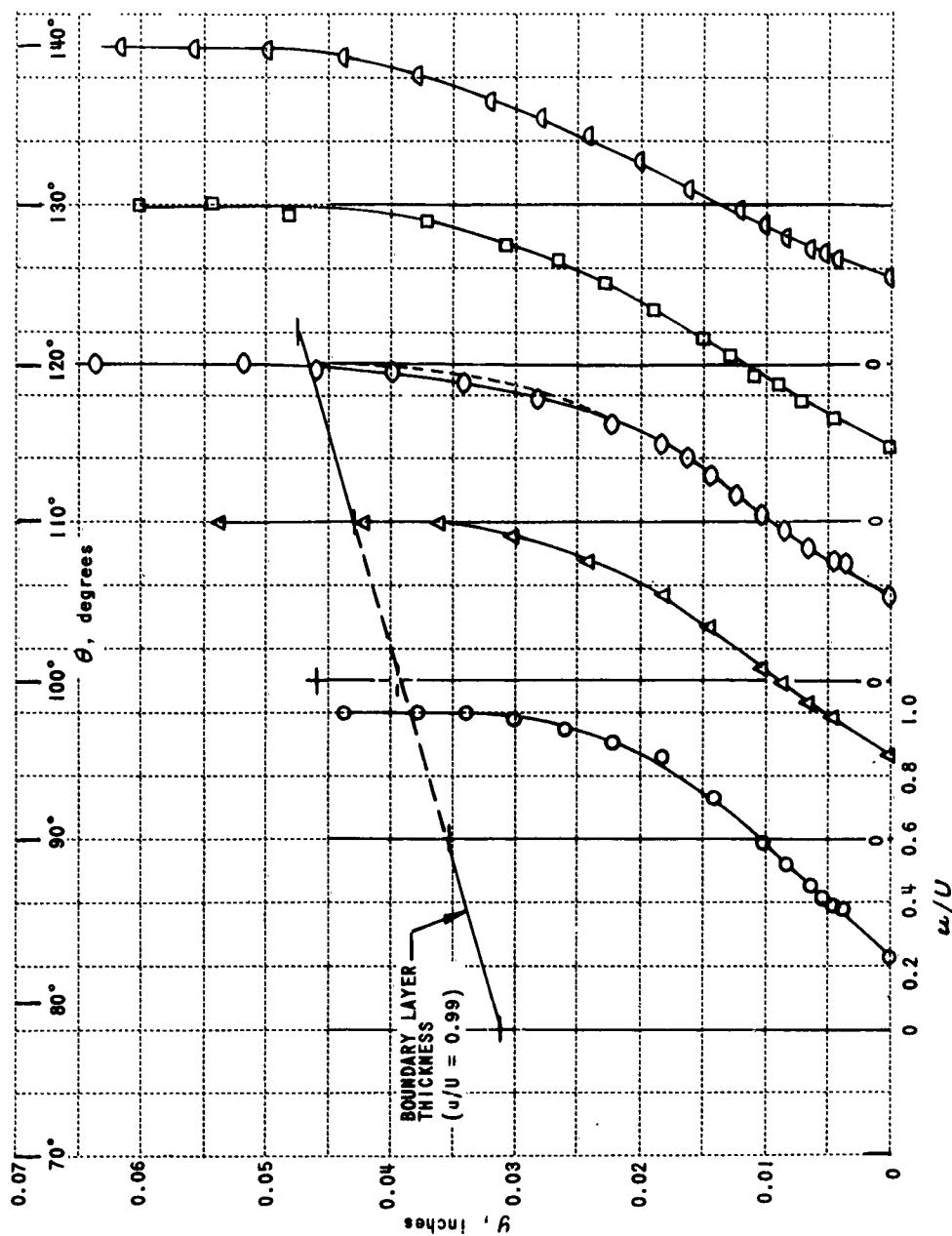


Figure 5 BOUNDARY-LAYER VELOCITY PROFILES AND THICKNESSES vs  $\theta$   
 $U_{\infty} \approx 30$  fps,  $\Omega \approx +500$  RPM

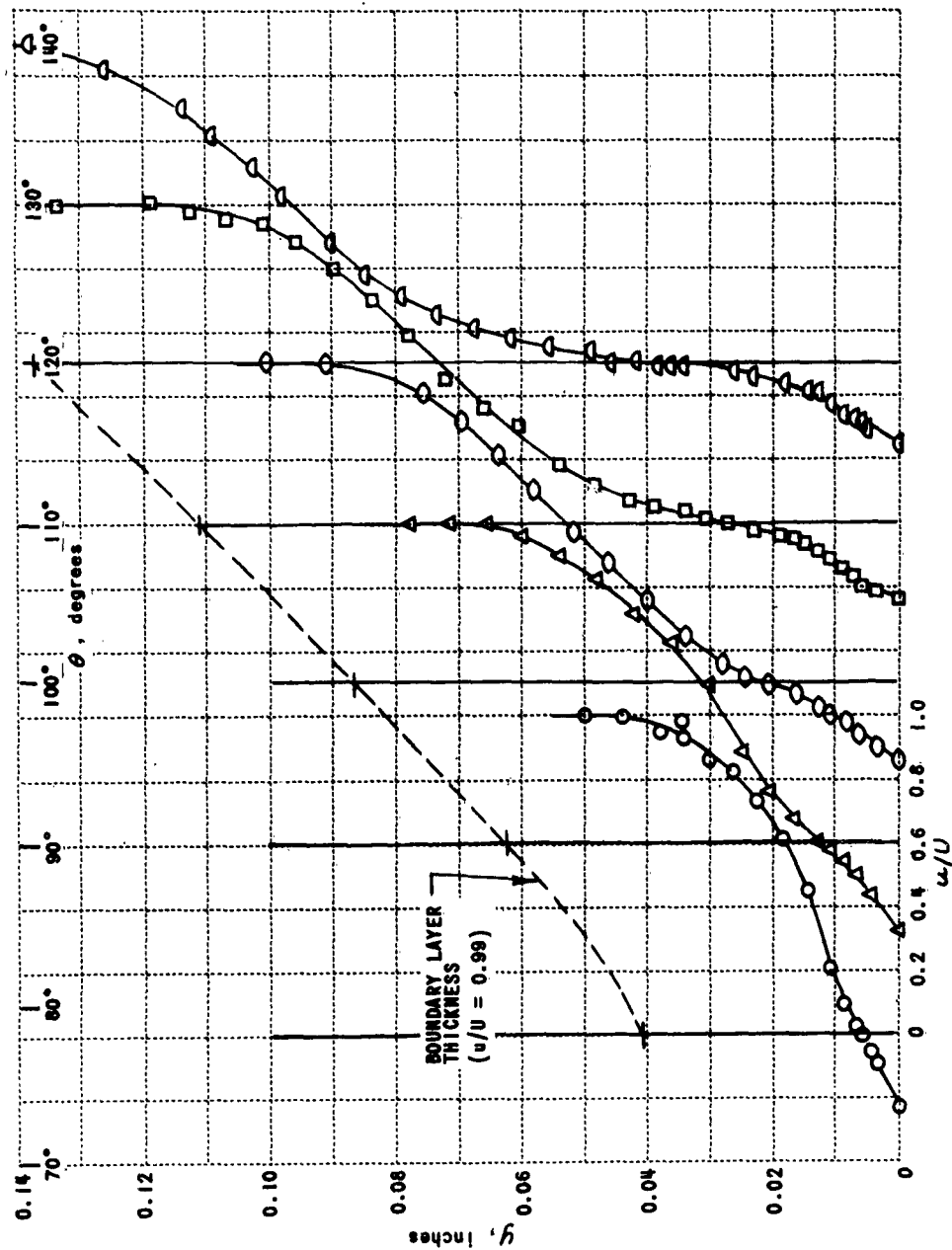


Figure 6 BOUNDARY-LAYER VELOCITY PROFILES AND THICKNESSES vs  $\theta$   
 $U_\infty \approx 30$  fps,  $\Omega \approx 500$  RPM



separation by the accepted standard for zero wall velocity. The profiles at large values of  $\theta$  appear to be flattening against the  $y$ -axis as hypothesized in Reference 2, but the abrupt change from a predicted relatively large flattened portion near the wall to the wall velocity at the wall is not apparent. The profiles appear to approach the wall velocity in a fairly gradual fashion with decreasing distance from the wall. The behavior of the profiles shown here and the gradual increase in the boundary-layer thickness implied by the smoke pictures of Reference 3 suggest that separation does not occur at a well-defined point but is, rather, a quite gradual process. Experimental data to be obtained at values of  $\theta$  greater than  $120^\circ$  should help to clarify this idea.

## B. Theoretical Results

The initial theoretical effort during this program is concerned with a continuation of the work reported in Reference 4. This work was a study of possible disturbance patterns which may exist in the flow field produced by a single-compressor stage -- with a view to the description of possible three-dimensional or partial-span modes.

It should be made clear that this theoretical study is not directed toward attempts to predict specific characteristics of rotating stall phenomena, such as speed of propagation, number of stall zones, or the prediction of when rotating stall will occur in terms of compressor characteristics. Rather, the investigation is concerned with compatibility conditions in the flow which determine whether rotating stall is a permissible disturbance of the particular type of flow considered.

Moore (Reference 4) showed that for small-disturbance axial flow with mean swirl of the free-vortex type through an axial compressor disk, the only type of rotating circulation disturbance on the compressor disk that could exist is one independent of blade radius which is fixed and rotating with the blades; for this type of disturbance, the resulting unsteady flow downstream of the compressor was irrotational. The theoretical work of the present program is an attempt to extend Moore's analysis to more general types of swirl in the steady flow upstream of the compressor disk. It is shown that if the radial component of disturbance velocity is zero (or more generally if the derivative of this component with respect to axial distance is zero) for any mean-flow swirl the only rotating circulation disturbance that can exist on the blades must rotate with the blades if the resulting disturbance flow is to remain bounded with both increasing time and with axial distance downstream of the compressor. (Moore's results apply whether the disturbance flow remains bounded or not.) Under these conditions the resultant unsteady flow is irrotational.

In particular, it would appear that these results would be applicable to flow through an axial compressor with a narrow annulus where it is reasonable to expect that radial disturbance velocities would be very small.

Linearized equations for the convection of vorticity were developed in References 4 and 1, for a cylindrical coordinate system with origin at

the center of the compressor disk:  $x$  is the axial coordinate, positive downstream;  $r$  is the radial coordinate, and  $\theta$  is the angular coordinate. These equations are

$$\left(\frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x} + \frac{U_0 s}{r} \frac{\partial}{\partial \theta}\right) \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix} \approx \begin{Bmatrix} -\left[\frac{U_0}{r}(rs)_r\right]_r v + \frac{U_0}{r}(rs)_r \frac{\partial u}{\partial x} \\ \frac{U_0}{r}(rs)_r \frac{\partial v}{\partial x} \\ \frac{U_0}{r} \left\{ (rs)_r \frac{\partial w}{\partial x} + [(rs)_r - s] \eta \right\} \end{Bmatrix} \quad (1)$$

where  $U_0$  is the free stream mean axial velocity,

$U_0 s$  is the tangential mean velocity,

$(\xi, \eta, \zeta)$  are the perturbation components of vorticity, and

$(u, v, w)$  are the perturbation velocity components.

The perturbation vorticity components are related to the perturbation velocity components by

$$\left. \begin{aligned} \xi &= \frac{1}{r} \frac{\partial(rw)}{\partial r} - \frac{1}{r} \frac{\partial v}{\partial \theta} \\ \eta &= \frac{1}{r} \frac{\partial u}{\partial \theta} - \frac{\partial w}{\partial x} \\ \zeta &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial r} \end{aligned} \right\} \quad (2)$$

Also, the continuity equation is

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial(rv)}{\partial r} + \frac{1}{r} \frac{\partial w}{\partial \theta} = 0 \quad (3)$$

These equations are subject to the requirement of periodicity with respect to  $\theta$  in the velocity and vorticity components.

If it is assumed that  $v = 0$ , Equations (1) become

$$\left(\frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x} + \frac{U_0 s}{r} \frac{\partial}{\partial \theta}\right) \begin{Bmatrix} \xi \\ \eta \\ \zeta \end{Bmatrix} = \begin{Bmatrix} \frac{U_0}{r} (rs)_r \frac{\partial u}{\partial x} \\ 0 \\ \frac{U_0}{r} \left[ \frac{\partial(rs)}{\partial r} \frac{\partial w}{\partial x} + \left[ \frac{\partial(rs)}{\partial r} - s \right] \eta \right] \end{Bmatrix} \quad (4)$$

where

$$\begin{aligned} \xi &= \frac{1}{r} \frac{\partial(rw)}{\partial r} \\ \eta &= \frac{1}{r} \frac{\partial u}{\partial \theta} - \frac{\partial w}{\partial x} \\ \zeta &= -\frac{\partial u}{\partial r} \end{aligned} \quad (5)$$

and Equation (3) becomes

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial w}{\partial \theta} = 0 \quad (6)$$

A solution for  $\eta$  is then

$$\eta(x, r, \theta, t) = g(\lambda, r) \quad (7)$$

where

$$\lambda = \theta - \left(s + \frac{s}{r}\right)x + sU_0 t$$

and  $sU_0$  is the rotational velocity of the circulation disturbance on the compressor disk. Note that periodicity in  $\theta$  for  $\eta$  requires that  $g(\lambda, r)$  be periodic in  $\lambda$ .

Consider next

$$\frac{\partial \xi}{\partial t} + U_0 \frac{\partial \xi}{\partial x} + \frac{U_0 s}{r} \frac{\partial \xi}{\partial \theta} = \frac{U_0}{r} \left\{ \frac{\partial(rs)}{\partial r} \frac{\partial w}{\partial x} + \left[ \frac{\partial(rs)}{\partial r} - s \right] g(\lambda, r) \right\} \quad (8)$$

from Equations (4). In particular, consider the particular integral which satisfies the nonhomogeneous term  $\frac{U_0}{r} \left[ \frac{\partial(rs)}{\partial r} - s \right] g(\lambda, r)$ . Assume this particular integral has the form  $\frac{U_0}{r} [a(x, r, \theta, t)] g(\lambda, r)$ .

Substituting in the left-hand side of Equation (8), and equating with  $\frac{U_0}{r} \left[ \frac{\partial(rs)}{\partial r} - s \right] g(\lambda, r)$  there results

$$a = a_0(\lambda, r) + \left[ \frac{\partial(rs)}{\partial r} - s \right] [(1 - a_0, U_0)t + a_1 x] \quad (9)$$

The function  $a_0(\lambda, r)g(\lambda, r)$  can be incorporated into the complementary solution of Equation (8). In Equation (9), there is only one arbitrary constant; hence, it can be required that  $\zeta$  be bounded as  $x$  becomes large,

or as  $t$  becomes large, but not both. Hence, if  $\xi$  is to be bounded for both  $x$  and  $t$ , it must be required that  $g(\lambda, r) = \eta = 0$ .

From Reference 4, Equation (45), the disturbance circulation  $\gamma$  is related to the appearance of radial vorticity at the compressor disk by

$$\frac{\partial \gamma}{\partial t} + k U_o \frac{\partial \gamma}{\partial \theta} = U_o r g_+ \quad (10)$$

where  $k U_o$  is the angular velocity of the blades and  $g_+ = g_{x=0}$ . Hence, if  $g = 0$ , then

$$\gamma(r, \theta, t) = H(\theta - k U_o t) + F(r) \quad (11)$$

only. Therefore, for bounded small perturbation flows with swirl a general function of radius in the mean flow, the only rotating circulation disturbance possible is one fixed in the blades, if  $v = 0$ ; note that this is true if it is only required that  $\partial v / \partial x = 0$ , i. e., that  $v$  is not a function of  $x$ .

At the compressor disk,

$$\begin{aligned} \frac{\partial \gamma}{\partial r} &= r \sqrt{\xi_+^2 + \zeta_+^2} \\ &= r \sqrt{1 + S^2 \xi_+^2} = F'(r) \end{aligned}$$

as

$$\zeta_+ - S \xi_+ = 0,$$

(Reference 4, Equations (46) and (38).) Hence, the unsteady small disturbance flow downstream of the compressor is irrotational, as the flow upstream is irrotational, and the unsteady circulation disturbance results in no vorticity shed into the flow downstream.

It is, of course, possible that even if the requirement of boundedness for the small disturbance flow had been relaxed, the same conclusion would have been reached, as was the case for the free-vortex swirl flow treated in Reference 4; this case has not as yet been treated in the current analysis.

#### IV WORK FOR THE NEXT QUARTER

##### A. Experimental Program

It is expected that the rotating cylinder experimental work will be concluded during the next quarter. A heavy workload is scheduled for the CAL One-Foot, High-Speed Wind Tunnel in which this work is being performed. It is desirable, therefore, to complete our tests as soon as possible to avoid excessive delays and cost associated with disassembly and reassembly of the apparatus that might result from fitting our tests into the tunnel schedule.

##### B. Theoretical Program

Work will continue on the compatibility analysis currently underway. Effort will initially be directed toward extending the analysis by removing the restriction that  $\partial v / \partial x$  be assumed zero.

##### C. General

Consideration will be given to the most desirable areas for future work on the program.

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